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A Discrete Optimization Model to Minimize Organ Recovery Time Using Heuristic Algorithms

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Abstract: This study proposes a discrete optimization model to minimize the organ recovery time in an Organ Procurement Organization (OPO) by grouping its associated hospitals and transplant centers into several clusters, based on their available organ recovery groups. Typically, the OPO covers a relatively large geographical area to recover organs from donors and deliver them to the recipients. Organs and/or tissues need to be transplanted within their viable time. Therefore, a discrete optimization model is proposed, based on the *p*-median approach to identify optimal locations of the organ recovery groups to recover the organs within a desired time interval. Three heuristic solution approaches, such as Multi-start Fast Interchange (MFI), Simulated Annealing (SA), and Lagrangian Relaxation Algorithm (LRA), are applied to solve the *p*-median clustering problems. Numerical examples are tested to identify a better solution approach in terms of a set of key performance indicators, such as elapse time, Silhouette index, and objective function value. The experimental results indicate that the MFI approach is effective finding an initial solution in the shortest possible time. To find a non-dominant optimal solution, the LRA outperformed the initial solution. In the future, the experimental results will be compared with real data to ensure the effectiveness of the proposed model.

Keywords: *p*-median, Multi-start fast interchange, Simulated annealing, Lagrangian relaxation

1. Introduction

The increasing success rate in organ transplantation has led to the growth in the number of waiting candidates for organ transplantation. In the United States, the United Network for Organ Sharing (UNOS) keeps track of the donor and recipient candidates, and they claim that more than 114,000 patients are waiting for organ transplantation. According to their records, a new name is added to the waiting list every 10 minutes. On the other hand, the organ supply is below the expected level. As a result, 18 people on average die every day while waiting for an organ in the U.S. In New York State, only 20% of the population over 18 years old have been registered for the donation, while it is 44% nationwide (New York Organ Donor Network, 2013). Therefore, the effective utilization of donated organs needs to be ensured to reduce the organ shortage between supply and demand in the New York State.

The organ allocation system follows a three-tier hierarchical system where the U.S. is divided into 11 geographic regions; i.e., these regions are further divided into 59 Organ Procurement Organizations (OPOs). Typically, in the New York metropolitan area, an OPO needs to serve a culturally and ethnically diverse population of more than 10 million people and work closely with many transplant centers and hospitals to coordinate organ, tissue, and eye donations for transplantations in their region (U.S. Department of Health and Human Services, 2013).

Two main allocation systems have been developed by the UNOS in the past, such as (1) allocating organs to patients with higher priority at the same locale, and (2) allocating organs to patients with the greatest medical need regardless of their locations. In the current practice, a procured organ is offered first at the local level, then regionally and nationally (Kong, 2005). The purpose of giving allocation priority to the local level is to increase the donor-recipient transplantation efficiency by reducing the organ transportation time, which ultimately reduces the Cold Ischemia Time (CIT). It has been reported that the duration of dialysis is another risk factor for graft and patient survival (Wolfe et al., 1999; Meier-Kriesche et al., 2000). Therefore, systematic coordination among the OPOs, transplant centers, and hospitals is necessary to identify the concerns regarding procurement, allocation, and transplantation of a donated organ under the geographic area of a particular OPO. However, there is ambiguity over effective collection of organs under regional configuration. In addition, when an organ is harvested, it starts losing its viability although the viability rate is organ-specific and its measured unit is the CIT. The CIT is defined as the time lag between the harvest and transplantation of an organ. Thus, it reduces organ viability and transplantation success rate. The Center for Organ Recovery and Education (CORE, 2013) claims that except corneas, the maximum viable time limit for most organs ranges between four and 24 hours. On the other hand, for the OPO in the New York Metropolitan area, the average time to harvest an organ and supply it to the operation room is around 30 hours. Hence, this is considered as significant dissatisfaction from customers' point of view. In this situation, an optimal process for organ harvesting needs to be developed to streamline workflow and improve transplantation success rate. Therefore, this study proposes a discrete optimization model for effective recovery and allocation of a recovered organ within the feasible time limit at the same locale.

The remainder of this study is structured as follows: A brief review of literature is presented in Section 2. The proposed discrete optimization model based on p-median algorithm and its solution by applying the Multi-start Fast Interchange (MFI), Simulated Annealing (SA), and Lagrangian Relaxation Algorithm (LRA) heuristics are depicted in Section 3. Experimental results are presented in Section 4. Finally, conclusions and future directions of the research are addressed in Section 5.

2. Literature Review

Organ allocation is a complex issue as many factors are related. Different organs have different specifications for organ allocation. Those specifications are blood type matching, similarity index, CIT, and others. Most research on organ transplantation focuses on policies that describe how to allocate organs for waiting patients (Rais and Viana, 2010). Markov chain, Analytical Hierarchy Process (AHP), Data mining, Multi-agents, Fuzzy logic, Simulation modeling, Data mining, and Hybrid algorithms are noteworthy algorithms in solving allocation problems. In addition, several efforts have been made to evaluate and compare different allocation methods for transplantations. A knowledge-based model was proposed to find an optimal donor-recipient selecting and matching decision by applying fuzzy techniques and AHP (Saha et al., 2012).

Organ viable times for different organs are shown in Table 1. It is also important to pay attention to improving the delivery of the harvested organs within the feasible time limit. The organ delivery system can be improved by grouping the hospitals and transplant centers into several clusters based on the available organ recovery groups. To cluster and optimize the problem, hospitals and transplant centers can be considered as a data set for p-median algorithm.

Table 1: Maximum Viable Time of Organs (CORE, 2013)

Organs	Viable time
Hearts and lungs	4 hours - 6 hours
Pancreas	12 hours - 24 hours
Livers	18 hours - 24 hours
Kidneys	48 hours - 72 hours
Corneas	Can be preserved 7-14 days, though preferred to use within 3 days

The p-median problem is one of the most studied issues in combinatorial optimization, having many applications, among which cluster analysis, various location problems, and optimal diversity management can be mentioned (Hansen and Jaumard, 1997; Mulvey and Crowder, 1979; Briant and Naddef, 2004). p-median problem was proved as NP-hard by Kariv and Hakimi (1979), and several heuristic algorithms have been found to solve the problem in an approximate way. Excellent coverage of heuristic methods for solving the p-median problem is provided by Mladenovic et al. (2007). Kaufman and Rousseeuw (1990) have provided an in-depth study on p-median (i.e., partitioning around centroids) methods for clustering. Although tremendous progress has been made in the development of exact solution methods for the p-median problems, the focus of their study is restricted to heuristic methods (Beltran et al. 2006). Different approximations and meta-heuristics designs were surveyed and it was found that most of them are based on interchange or swap-based local search (Reese, 2006; Alba and Dominguez, 2006).

The increasing number of solution methods and the availability of powerful computers have contributed enormously to solve facility location problem. In this study, a facility location problem is taken into consideration with respect to hospitals, transplant centers, and organ recovery groups. p objects from these hospitals and transplant centers are considered as medians The goal is to choose the location of these medians (i.e., organ recovery groups) and assign all objects (i.e., hospitals and transplant centers) to their nearest median with the objective of minimizing the sum of the distances between the medians and objects assigned to their cluster.

3. Methodology

To develop a discrete optimization model, the hospitals and transplant centers are put into several clusters where the number of clusters depends on the number of organ recovery groups available for a particular period of time. If a group is assigned to serve a hospital to recover organs, the hospitals and transplant centers are regrouped based on the available number of recovery groups. Several assumptions have been made for this study: (a) organs are offered to the patients who live under the coverage area of an OPO (same locale); (b) distance among the hospitals and transplant centers and the number of organ recovery groups are considered as decision-making parameters; and (c) allocation of organs for the patients from other OPOs is not considered. The p-median cluster model is a facility location model and the number of facilities or recovery centers to be opened or clusters to be formed (p) is determined by a priori information. On the basis of a distance matrix, $\boldsymbol{D}$, several elements are selected in which the rest of the elements are allocated, such that p number of clusters that can be created. The procedure is repeated when the number of available organ recovery group changes. Figure 1 shows the structural design of the proposed discrete optimization model.

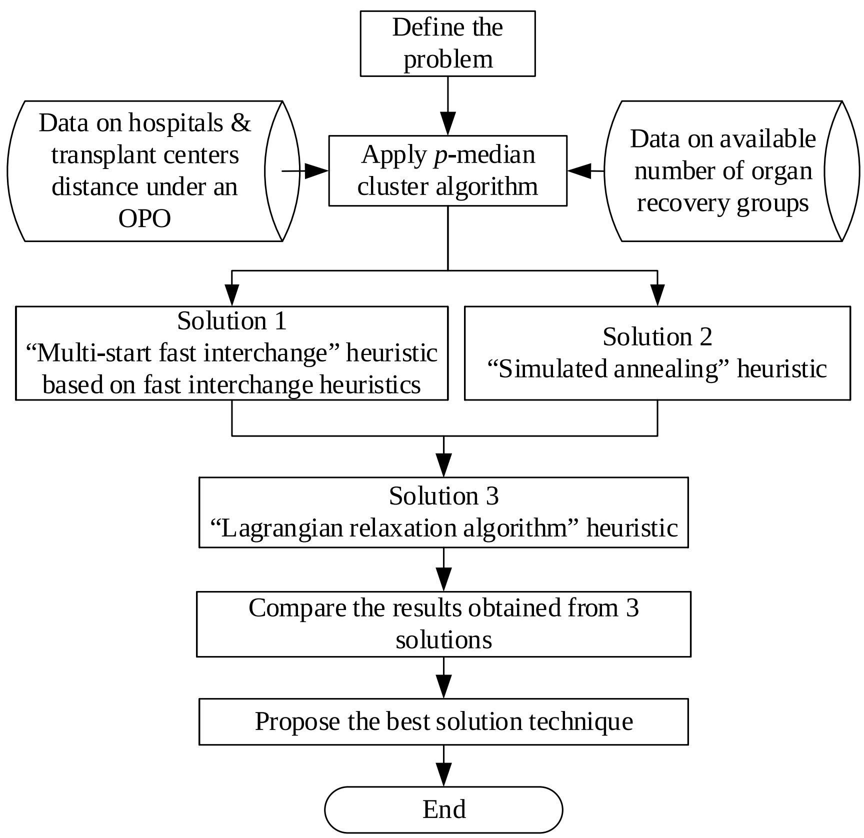

Figure 1: Structural Design of the Proposed Model

A cluster median is defined as the element j that is representative of all elements in the cluster. As the objective is to create p clusters, so there will be p-medians with $1 \leq p \leq m$ where m is the number of products that has to be clustered. The variables are defined as follows: $x_{ij} = 1$, element i is allocated to median j; 0, otherwise; $y_j = 1$, element j is a median; 0, otherwise; d_{ij} = distance, time or cost between location i and j; I is the set of elements that have to be clustered; J is the set of elements that can act as medians. The p-median facility location model can be formulated as follows:

$$min \sum_{i \in I} \sum_{j \in J} d_{ij}\, x_{ij}, \forall\, d_{ij} \in \boldsymbol{D} \tag{1}$$

$$\text{s.t.} \sum_{j \in J} x_{ij} \;=\; 1, \forall\, i \in I \tag{2}$$

$$\sum_{j \in J} y_j \;=\; p \tag{3}$$

$$x_{ij} \;\leq\; y_j, \forall\, i \in I, \forall\, j \in J \tag{4}$$

$$x_{ij}, y_j \;\in\; \{0,1\}, \forall\, i \in I, \forall\, j \in J \tag{5}$$

In this study, the $\boldsymbol{D}$ matrix measures performance. This matrix provides an indication of the similarity between every two elements with respect to a certain criterion. The distance matrix $\boldsymbol{D}$ is a $(m \times n)$ symmetric matrix of d_{ij} between all pairs of vertices i and j. The vertex median p is that vertex for which the sum of the elements in the corresponding column of $\boldsymbol{D}$ is minimized.The objects/data for this study are quantitative in nature. These data can be stored in a $(n \times d)$matrix, where n and d are the number of objects (i.e., hospitals and transplant centers) and variables (i.e., distances) respectively.

The proposed optimization model distributes the hospitals and transplant centers under an OPO into several clusters in such a way that it increases the quality of performance by reducing its response time. To achieve this goal, data about the location of hospitals, transplant centers, and manpower strength of an OPO need to be collected. Then, algorithms are proposed based on three different solution techniques to solve the p-median algorithm problem, and finally compare the solution approaches to obtain the best solution. It is crucial to locate p distinct points (number of organ recovery groups) in the search space, such that data points within a cluster are more similar than data points in other clusters. Then, all the n data points are divided into p groups and centroids for these p clusters are real objects and these objects serve as exemplars (i.e., locations of the selected organ recovery groups) for their clusters. Three different approaches, namely, MFI, SA, and LRA heuristics are applied to solve the problem, and finally results are compared to propose a better solution approach among them.

3.1 Multi-start Fast Interchange Heuristic

The Multi-start heuristic was first proposed by Whitaker in 1983, but uses of the interchange rules were first described by Hansen and Mladenovic (Whitaker, 1983; Hansen and Mladenoviĉ, 1997). In this approach, the problem is initially solved by Greedy heuristic, the obtained solution is applied as the initial solution for the Fast Interchange heuristic, and then, the obtained solution is used as the initial solution for all methods tested (i.e., if the solution is obtained by Greedy or Greedy plus Fast Interchange, then no comparison is required). Hansen and Mladenoviĉ (1997) claimed that the search through variable neighborhood search is an attractive area and this approach is better than Tabu search. Kaufman and Rousseeuw (1990) proposed that the average Silhouette index could be applied for evaluation of clustering validity as well as to decide how sound the number of selected clusters is. The Silhouette index can be defined as:

$$S(i) = \frac{\{b(i) - a(i)\}}{max\{a(i), b(i)\}} \tag{6}$$

where $a(i)$ is average dissimilarity of i-th object to all other objects in the same cluster and $b(i)$ is the minimum of average dissimilarity of i-th object to all objects in another cluster (in the closest cluster). It follows the formula $-1 \leq S(i) \leq 1$ and if $S(i)$ value is close to 1, it means that sample is “well-clustered” and assigned to an appropriate cluster.

3.2 Simulated Annealing Heuristic

The Affinity Propagation (AP) procedure is capable of obtaining reasonable solutions in a modest computation time. However, Brusco and Köhn (2009) found that affinity propagation seldom obtains a global optimal solution for small data sets as well as to some extent for large data matrices. Therefore, the SA approach is applied to overcome the obstacles of the AP. If the number of objects exceeds 2000, the cooling parameter value can be in a range between 0.8 and 0.9. Increasing the cooling parameter value from 0.8 to 0.9 can increase the solution quality as well as the computational time (Brusco and Köhn, 2009).

3.3 Lagrangian Relaxation Algorithm Heuristic

This is a three-stage method consisting of a Greedy heuristic, Lagrangian relaxation, and a Branch-and-bound algorithm to produce a globally optimal solution for a p-median problem (Brusco and Köhn, 2009). To find the largest lower bound, the algorithm searches for a vector λ that maximizes the objective function. The objective function is as follows:

$$max_{\lambda}[min_{X,Y}\, Z_2(\lambda) = \sum_{i=1}^{N}\lambda_i + \sum_{i=1}^{N}\sum_{j=1}^{N}(d_{ij}\lambda_i)x_{ij}] \quad (7)$$

All the solution algorithms are developed by using MatLab software and the solutions are compared based on their objective function value and solution time to propose a better approach to solve the p-median problem. In the next section, discussion is made on the analysis of the experimental results obtained from the MatLab software.

4. Experimental Results

The effectiveness of the solution techniques is compared by varying the input parameters into several levels to observe the output of each solution technique. The output performance measures include exemplars, objective function, and elapse time for calculation. In addition, for the MFI and SA techniques, the $S(i)$ indicates the perfection for a number of objects in a cluster for that particular technique. For the LRA, optimal solutions are achieved for $Terminate$=1 and E=0, otherwise, $Terminate$=0 and E is some positive value. Therefore, for the experiments, K=2, 3, 4; $Restart$=20, 35, 50 (i.e., number of iterations of the solution algorithm); $Cool_{SA}$=0.6, 0.75, 0.9; and upper bound of the objective function (ZUB) take the best-found lower bound of the objective function value obtained from the MFI approach for the same value of K. The output measures are analyzed based on the parameter settings as shown in Table 2.

Table 2: Parameter Settings for Experiments

Parameters	Values
K	2, 3 or 4
Restarts	20, 35 or 50
$Cool_{SA}$	0.60, 0.75 or 0.90
ZUB	16,392, 10,631 or 8,129

4.1 MFI, SA, and LRA Solution Approach

The MFI solution approach is applied on the p-median model based on nine different combinations of K and $Restarts$. It is observed that for similar values of K, irrespective of any combination with $Restarts$, exemplars and Objective Function of MFI (Z_{MFI}) values do not change. The $S(i)$ values for K= 2, 3, and 4 are 0.62, 0.59, and 0.57, respectively. The SA approach is also applied on that model based on different combinations of K and $Cool_{SA}$ for nine alternatives. It is observed that for the similar K values, Objective Function of SA (Z_{SA}) values do not change, however, exemplars value changes with the change of $Cool_{SA}$ value, which is different than the MFI approach. The $S(i)$ values for K= 2, 3, and 4 are 0.62, 0.59, and 0.57, respectively which is same as the MFI.

Nine experiments are conducted on the p-median model for both MFI and SA solution approaches, and three objective function values are obtained for different combinations of parameters settings. Figure 2 shows the objective function and $S(i)$ values of the MFI for nine different combinations of parameter settings. The SA also shows similar objective function and $S(i)$ values for the combinations of its parameter settings. These objective function values are considered as input parameter ZUB for the LRA approach to obtain Lower Bound of the Objective Function (ZLB). The p-median model is also run for nine different combinations of parameters settings based on the LRA solution approach. Table 3 presents the objective function values of the LRA for nine different parameter settings. It is observed that the optimal solution values of the MFI and SA approaches for the same combination of K and $Restarts$ or $Cool_{SA}$ value are also the optimal function value of the LRA approach. Therefore, the LRA approach can help in justifying the optimal value of the objective function, and it is effective in determining the lower bound of the objective function. However, before applying this solution approach, the MFI or SA approach is run and the resulting objective function values from these procedures are considered as the ZUB. Similar to the SA approach, exemplars value also changes with the changing of $K$and ZUB. Optimal solution of the problem is achieved for the parameter settings combination no. 1, 2, 3, 5, and 6 as $Terminate$ and E values are 1 and 0, respectively.

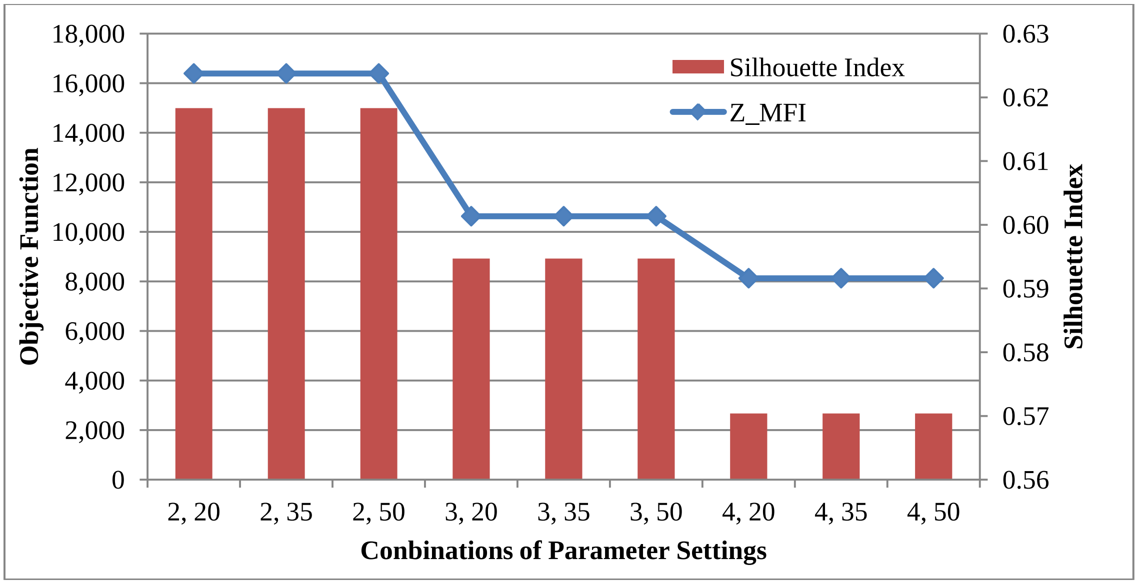

Figure 2: Objective Function and Silhouette Index Values of the MFI Approach

Table 3: Objective Function Values of the LRA for Different Combinations of Parameter Settings

Combination No.	Parameter Settings (K, ZUB)	ZLB
1	2, 16,392.00	16392.00
2	2, 10,631.00	13429.00
3	2, 8,129.40	11545.00
4	3, 16,392.00	10631.00
5	3, 10,631.00	10631.00
6	3, 8,129.40	9188.20
7	4, 16,392.00	8129.30
8	4, 10631.00	8129.30
9	4, 8129.40	8129.30

4.2 Comparison among the MFI, SA and LRA Solution Approach

Both MFI and SA solution methods tend to be extremely effective for usually five or fewer clusters, and certainly no more than 10 clusters. The significant performance differences between MFI and SA occur when the number of clusters increases to more than 10, particularly when K increases beyond 50. However, in this study, the maximum number of clusters is four. Therefore, both methods are equally effective. The MFI heuristic can run multiple restarts, whereas the SA heuristic is set for a single restart due to its substantially higher computation time. Elapse time for three different heuristics are shown in Table 4 and it can be observed that same optimal objective function values are obtained for both MFI and SA approaches; however, the computational time for the SA is almost 95% higher than the MFI, which is a significant drawback of the SA approach. On the other hand, the computational time for the LRA approach is around 88% higher than the MFI approach, but 54% less than the SA approach. From the above discussions, it can be claimed that the MFI approach is better than the SA approach in achieving initial solution in the shortest possible time. To obtain the lower bound of the objective function, the LRA approach can be applied on the initial solution.

Table 4: Elapse Time for the MFI, SA, and LRA Solution Approaches

Combination No.	MFI Parameter Settings		MFI Elapse Time (s)	SA Parameter Settings		SA Elapse Time (s)	LRA Parameter Settings		LRA Elapse Time (s)
	K	Restarts		K	$Cool_{SA}$		K	ZUB	
1	2	20	9.49	2	0.60	189.28	2	16392.00	29.36
2	2	35	17.96	2	0.75	316.12	2	10631.00	0.12
3	2	50	24.25	2	0.90	849.94	2	8129.40	0.12
4	3	20	13.96	3	0.60	209.64	3	16392.00	540.59
5	3	35	25.50	3	0.75	357.92	3	10631.00	35.53
6	3	50	36.21	3	0.90	939.61	3	8129.40	0.13
7	4	20	23.91	4	0.60	280.67	4	16392.00	528.53
8	4	35	40.60	4	0.75	468.92	4	10631.00	527.34
9	4	50	58.23	4	0.90	1262.88	4	8129.40	539.76

5. Conclusions and Future Works

The objective of this study was to develop a discrete dynamic optimization model to group the hospitals and transplant centers into several clusters based on the number of available organ recovery groups at a particular point of time. The proposed model is formulated with *p*-median approach, and the MFI, SA, and LRA heuristics are considered as the solution approaches. The key outcomes from the experimental results based on the key performance indicators are (1) the MFI approach provides better initial solution than the SA approach, (2) the LRA approach provides the best optimal solution for obtaining the lower bound of the objective function, and (3) the MFI approach takes minimum elapse time to obtain exemplars. Therefore, it is well established that to obtain optimal location of the organ recovery group, the MFI approach should be applied first to obtain an initial solution and the LRA approach can be applied to obtain the non-dominant optimal solution. In the future, this study can be conducted with real and larger data set. Verification and validation of the experimental results are important issues. For this, the experimental results need to be implemented in any OPO to analyze the credibility and effectiveness of this study. In addition, CIT of different organs can also be considered while clustering the hospitals and transplant centers. It is expected that implementation of this result in OPOs will bring efficiency in their performance level by delivering organs within the viable time limit.

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